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Critical properties of a spin glass with anisotropic Dzyaloshinskii–Moriya interaction

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Abstract. We study the classical n-vector ($n \geqslant 3$) spin glass with *anisotropic* quenched random Dzyaloshinskii–Moriya (DM) interaction. A random DM interaction with m independent and separated couplings defines a generalized gauge-glass model with a $O(n-2m) \otimes^m O(2)$ rotational symmetry and a broken global reflection invariance. It is shown that this model is in the same universality class as the random-gauge XY ('gauge glass') model. With an additional uniaxial anisotropy, a crossover from Ising-like to gauge-glass critical behaviour is found for a sufficiently large variance of the DM interaction. A new situation arises when there is correlation between the separated random DM couplings. We show that the critical behaviour of a spin glass with two *correlated* couplings of the anisotropic DM interaction is in a new universality class. The critical exponents η and ν of this model are calculated at two-loop order near six dimensions. We also present a simplified and more rigorous field-theoretic analysis of the gauge-glass model.

1. Introduction

In the theory of spin glasses [1, 2] the Dzyaloshinskii–Moriya (DM) interaction, added to the usual isotropic exchange coupling, is well known as an important source of macroscopic anisotropy [3], since it breaks the global rotational invariance. Microscopically, the DM interaction is mediated by spin–orbit scattering of the conduction electrons at non-magnetic impurities. Its influence on the properties of spin glasses has been studied quite intensively [3–9]. In particular, it has been shown that a quenched random DM interaction with isotropic couplings causes a crossover to Ising-like critical behaviour in the n-vector spin glass [4, 5].

A relation has also been established between the XY spin glass with random DM interactions and the random-gauge XY ('gauge glass') model. The XY gauge glass [10, 11] is of particular interest in the theory of Josephson-junction arrays [12], of granular superconductors [13, 14], and of the vortex glass phase [15–17] in the high-T_c superconductors. Numerical real-space renormalization-group studies [18, 19] have shown that the random-bond XY spin glass with quenched random DM interaction and the XY gauge glass belong to the same universality class, which is different from those of all other isotropic n-vector spin glasses [20, 21]. This is also apparent in a field-theoretic analysis [22, 23], which shows that the critical behaviour of both the random-gauge XY model [22] and the random-bond XY model with DM interaction [23] is controlled by a two-component field, in contrast to a n^2-component field in the case of the isotropic n-vector spin glass. Such a reduction of the degeneracy of the eigenvalue which corresponds to the critical fluctuation mode has been attributed to the broken global reflection symmetry in these models [23, 24].

The possible role of the quenched local $O(2)$ gauge invariance for the value of the lower critical dimension is still under debate [19].

Since in the case of the XY spin glass (with $n = 2$) the addition of the DM interaction is of crucial importance for the critical properties and leads to a new universality class, one may suspect that a similar effect also occurs for the general n-vector spin glass upon introduction of a DM coupling. In the present paper we study two models which appear as natural candidates for a generalization of the peculiar symmetry resulting in the universality class of the gauge-glass model. A quenched random DM interaction without preferred orientation of the couplings leads to the critical behaviour of the Ising spin glass [4, 5] due to the lack of a global rotation symmetry. In section 2 we therefore consider the n-vector ($n \geqslant 3$) spin glass with an *anisotropic* DM coupling of neighbouring spins and with *no correlation* between the different separated components of the DM interaction. This results in a global $O(n-2m) \otimes^m O(2)$ rotational invariance, but destroys the reflection symmetry. To describe a possible competition between the DM interaction and the dipolar spin couplings, we will also allow for a uniaxial magnetic anisotropy. It is found that the critical behaviour of this generalized model is that of the random-gauge XY model. On the other hand, a sufficiently large uniaxial anisotropy results in Ising-like critical behaviour.

In section 3 we investigate the n-vector spin glass *with correlation* between separated couplings of the anisotropic DM interaction. It is shown that this peculiar type of symmetry leads to a new universality class. The corresponding critical exponents η and ν are calculated up to order $\mathcal{O}(\epsilon^2)$ near six dimensions, with $\epsilon = 6 - d$.

The field-theoretic formulation of Gingras for XY-type spin–glass models without global reflection symmetry [23] is strictly valid only for the *Gaussian* DM spin glass, since only the leading terms of the cumulant expansion have been considered. To show more clearly the relation between the random-gauge XY model and the XY spin glass with DM interaction, in section 4 we discuss a simplified and more rigorous field-theoretic treatment of the random-gauge XY spin glass.

2. Spin glass with uncorrelated separated DM interactions and uniaxial anisotropy

In this section we study the n-component ($n \geqslant 3$) vector spin glass with an anisotropic quenched random Dzyaloshinskii–Moriya interaction with separated couplings. The effect of a uniaxial magnetic anisotropy, which we choose in the direction of the nth spin component, will also be considered. The Hamiltonian of this model reads

$$\mathcal{H} = \sum_{\langle i,j\rangle} \sum_{ab} (J_{ij}(S_{ia}\Gamma_{ab}S_{jb}) + D_{ij}^{ab}(S_{ia}S_{jb})) \tag{1}$$

where the summation $\langle i, j\rangle$ is over the bonds of a simple cubic lattice, the S_{ia} ($a = 1, \ldots, n$) are the components of classical spins, and $\Gamma_{ab} = \delta_{ab} + \gamma\delta_{an}\delta_{bn}$ is the symmetric quadrupole tensor. In principle, besides the uniaxial anisotropy one could also have considered more general random dipolar couplings Γ_{ab}. However, additional diagonal or off-diagonal terms of the symmetric interaction Γ in (1) will, in general, drive the system to the critical behaviour of the Ising spin glass. Since we intend to study some new peculiar features of the antisymmetric DM interaction, we do not consider these dipolar terms here. The DM interaction with the couplings $D_{ij}^{ab} = -D_{ij}^{ba}$ is the generalization of the usual three-dimensional case, with the components a and b defining the plane in which the spins interact. Anisotropy of the DM interaction is introduced by the assumption that the couplings D_{ij}^{ab} are non-zero only for $m < n/2$ such planes. We also assume that the couplings are *separated*, i.e. these planes *do not intersect*. This requirement is crucial to find other than

Ising-like critical behaviour. If we disregard the uniaxial anisotropy, the DM interaction reduces the rotational symmetry of the Hamiltonian from $O(n)$ to $O(n-2m)\otimes^m O(2)$, and the interaction term related to D_{ij}^{ab} also breaks the global reflection invariance under $S_a \to -S_a$ or $S_b \to -S_b$. The quenched random couplings J_{ij} and D_{ij}^{ab} obey Gaussian probability distributions,

$$\mathcal{P}(J_{ij}) = \frac{1}{J\sqrt{2\pi}} \exp\left(-\frac{(J_{ij})^2}{2J^2}\right) \qquad \text{and} \qquad \mathcal{P}(D_{ij}^{ab}) = \frac{1}{D\sqrt{2\pi}} \exp\left(-\frac{(D_{ij}^{ab})^2}{2D^2}\right) \tag{2}$$

where all the non-zero $\langle (D_{ij}^{ab})^2 \rangle$ have no common component a or b. This is equivalent to the requirement that the planes of interaction do not intersect. In contrast to the situation studied in the next section, the D_{ij}^{ab} are uncorrelated, i.e. they are, independently of each other, described by Gaussian distributions.

To perform the quenched disorder average, we employ the replica method, introduce the spins S_{ia}^{α} with replica index $\alpha = 1, \ldots, n_r$, and obtain the replicated partition function

$$\begin{aligned}\langle Z^{n_r} \rangle = \mathrm{Tr}_{\{S\}} \exp\Big\{ \tfrac{1}{2} \sum_{\langle i,j \rangle} \sum_{\substack{ab \\ \alpha\beta}} \big((\beta J)^2 \Gamma_{aa} \Gamma_{bb} \; S_{ia}^{\alpha} S_{ib}^{\beta} S_{ja}^{\alpha} S_{jb}^{\beta} \\ + (\beta D)^2 d_{ab} \; (S_{ia}^{\alpha} S_{ia}^{\beta} S_{jb}^{\alpha} S_{jb}^{\beta} - S_{ia}^{\alpha} S_{ib}^{\beta} S_{jb}^{\alpha} S_{ja}^{\beta}) \big) \Big\} .\end{aligned} \tag{3}$$

Here we have used that Γ_{ab} is diagonal. We define $d_{ab} = 1$ if $\langle (D_{ij}^{ab})^2 \rangle \neq 0$, and $d_{ab} = 0$ otherwise. Next we introduce the spin-glass fields $Q_{iab}^{\alpha\beta} = S_{ia}^{\alpha} S_{ib}^{\beta}$. We omit quadrupole fields $Q_{iab}^{\alpha\alpha}$ since they generate only massive modes and do not couple to the soft modes which govern the critical behaviour. The trace in the partition function (3) can be performed explicitly after a Hubbard–Stratonovitch transformation, since only quartic products of spins are involved. However, we directly recall the probability distribution of the spin-glass fields [25],

$$\begin{aligned}\mathcal{P}(\{Q\}) &= \mathrm{Tr}_{\{S\}} \, \delta(Q_{iab}^{\alpha\beta} - S_{ia}^{\alpha} S_{ib}^{\beta}) / \, \mathrm{Tr}_{\{S\}}(1) \\ &\simeq \exp\Big(-\tfrac{1}{2} \sum_{\substack{ab \\ \alpha<\beta}} (Q_{iab}^{\alpha\beta})^2 + \sum_{\substack{abc \\ \alpha<\beta<\gamma}} Q_{iab}^{\alpha\beta} Q_{ibc}^{\beta\gamma} Q_{ica}^{\gamma\alpha} + \mathcal{O}(Q_i^4) \Big) .\end{aligned} \tag{4}$$

This immediately allows us to express $\langle Z^{n_r} \rangle$ in the Q-fields,

$$\begin{aligned}\langle Z^{n_r} \rangle = \int \mathcal{D}Q \; \exp\Big\{ \sum_{\substack{ab \\ \alpha<\beta}} \Big[-\tfrac{1}{2} \sum_i (Q_{iab}^{\alpha\beta})^2 + \sum_{\langle i,j \rangle} \Big((\beta J)^2 \Gamma_{aa} \Gamma_{bb} \; Q_{iab}^{\alpha\beta} Q_{jab}^{\alpha\beta} \\ + (\beta D)^2 d_{ab} \; (Q_{iaa}^{\alpha\beta} Q_{jbb}^{\alpha\beta} - Q_{iab}^{\alpha\beta} Q_{jba}^{\alpha\beta}) \Big) \Big] + \sum_{\substack{abc \\ \alpha<\beta<\gamma}} \sum_i Q_{iab}^{\alpha\beta} Q_{ibc}^{\beta\gamma} Q_{ica}^{\gamma\alpha} + \mathcal{O}(Q_i^4) \Big\} .\end{aligned} \tag{5}$$

To investigate the critical behaviour of the effective field theory (5) for the spin-glass order parameter, we have to identify the fluctuation modes which become soft at the phase transition. To extract the long-wavelength behaviour of the fields, we first introduce the momentum dependence via the Fourier transformation

$$Q_{ab}^{\alpha\beta}(\boldsymbol{k}) = \frac{1}{\sqrt{N}} \sum_i \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{r}_i) \; Q_{iab}^{\alpha\beta} \tag{6}$$

where N is the number of spins, and retain only the $\boldsymbol{k} = 0$ components of the Q-fields. Since the fluctuation modes which control the critical behaviour are determined by the quadratic terms in (5), we disregard higher powers of Q for the moment to obtain

$$\langle Z^{n_r}\rangle = \int \mathcal{D}Q \exp\Big\{-\tfrac{1}{2}\sum_{\substack{abcd\\ \alpha<\beta}} Q^{\alpha\beta}_{ab} M^{\alpha\beta}_{ab,cd} Q^{\alpha\beta}_{cd} + \mathcal{O}(Q^3)\Big\} \tag{7}$$

where $\boldsymbol{M}$ is the inverse zero-momentum propagator of the Q-fields. We find the eigenvalues λ of the $(n^2 \times n^2)$-matrix $\boldsymbol{M}$ and the corresponding degeneracies,

$$\begin{aligned}
&\lambda_1 = 1 - z(\beta J)^2 && [(n-1)^2 - 4m]\text{-fold degenerate}\\
&\lambda_2 = 1 - z\beta^2(J^2 + D^2) && 2m\text{-fold degenerate}\\
&\lambda_3 = 1 - z\beta^2(J^2 - D^2) && 2m\text{-fold degenerate}\\
&\lambda_4 = 1 - z(\beta J)^2(1+\gamma)^2 && \text{non-degenerate}\\
&\lambda_5 = 1 - z(\beta J)^2(1+\gamma) && 2(n-1)\text{-fold degenerate}\,.
\end{aligned} \tag{8}$$

Here z denotes the number of nearest neighbours in the (simple cubic) lattice. In order to identify the modes which become soft first, we note that for $D > 0$ the smallest eigenvalue must be either λ_2 or λ_4, depending on the value of the involved parameters. Therefore, the critical properties are determined by fluctuations described by either a one- or a $2m$-component field. While the former case results in the critical behaviour of the Ising spin glass [4, 25], the latter can be mapped on a cubic field theory with m non-interacting two-component fields, as is shown explicitly below. Such a cubic two-component field theory describes the critical properties of the XY gauge glass [22, 23]. Thus, the condition for XY gauge-glass critical behaviour reads

$$D^2/J^2 > \gamma(\gamma + 2)\,. \tag{9}$$

For vanishing DM coupling, $D = 0$, and in the range $-2 < \gamma < 0$, which corresponds to an easy-(hyper) plane anisotropy, the smallest eigenvalue $\lambda_1 = \lambda_2 = \lambda_3$ is $(n-1)^2$-fold degenerate. This corresponds to the critical behaviour of the isotropic $(n-1)$-component vector spin glass [25]. Other non-zero values of γ again lead to Ising-like behaviour which can be found for both positive and negative γ, since after disorder averaging the uniaxial anisotropy enters quadratically in λ_4. By inspection of (8), one may also find the mean-field critical temperatures $kT_{\mathrm{c}} = \sqrt{z(J^2 + D^2)}$ for the gauge-glass transition, and $kT_{\mathrm{c}} = \sqrt{z}J(1+\gamma)$ for the Ising-like case.

We now recover the full momentum dependence in the action of equation (5) and identify the critical fluctuation modes for the non-trivial case described by the inequality (9), when λ_2 is the smallest eigenvalue. For every pair of spin components (ab) (with $a < b$) for which $d_{ab} = 1$ we find two eigenmodes,

$$\mathrm{Re}\Psi^{\alpha\beta}_l = \zeta(Q^{\alpha\beta}_{aa} + Q^{\alpha\beta}_{bb}) \qquad \text{and} \qquad \mathrm{Im}\Psi^{\alpha\beta}_l = \zeta(Q^{\alpha\beta}_{ab} - Q^{\alpha\beta}_{ba}) \tag{10}$$

which define the m complex critical order parameter fields Ψ_l, where $l = 1, \ldots, m$. With the normalization factor ζ and the effective mass r,

$$\zeta = \left(a^2 \frac{1-\lambda_2}{2z}\right)^{1/2} \qquad \text{and} \qquad r = \frac{z\lambda_2}{1-\lambda_2} \tag{11}$$

where a is the lattice spacing, the Hamiltonian which describes the critical behaviour of the uncorrelated DM spin glass with separated couplings is finally obtained,

$$\mathcal{H}(\{\Psi_l\}_l) = \sum_{l=1}^{m} \mathcal{H}_l(\Psi_l) = \sum_{l=1}^{m}\Big\{\tfrac{1}{8}\sum_{\alpha\neq\beta}\big[\overline{\Psi}^{\alpha\beta}_l (r - \nabla^2)\Psi^{\alpha\beta}_l + \mathrm{CC}\big] + \frac{g_0}{3!}\,\mathrm{Tr}\,\Psi^3_l\Big\}\,. \tag{12}$$

This Hamiltonian corresponds to m decoupled two-component fields with cubic interaction and thus is in the same universality class as the random-gauge XY model. The decoupling of the critical fluctuation fields appears as a consequence of the reduced $O(n-2m)\otimes^m O(2)$ rotational symmetry of the magnetic Hamiltonian due to the anisotropic nature of the DM interactions. It is therefore this peculiar reduction of the symmetry which leads to the critical properties of the XY gauge glass also for random magnetic systems with more than two components.

3. Spin glass with correlated separated DM interactions

In this section we consider the n-vector spin glass with Gaussian anisotropic DM interaction, again with the assumption that all non-zero D_{ij}^{ab} have no common component a or b. However, we choose the D_{ij}^{ab} for different pairs (ab) of spin components *not independent* of each other. Instead, a correlation between the $m < n/2$ separated couplings D_{ij}^{ab} is introduced, which we describe by the Hamiltonian

$$\mathcal{H} = \sum_{\langle i,j\rangle}\left(J_{ij}\sum_{ab}(S_{ia}\Gamma_{ab}S_{jb}) + D_{ij}\sum_{(ab)}d_{ab}(S_{ia}S_{jb} - S_{ib}S_{ja})\right) \tag{13}$$

where we again allow for the uniaxial anisotropy Γ_{ab}, and the Gaussian probability distributions

$$\mathcal{P}(J_{ij}) = \frac{1}{J\sqrt{2\pi}}\exp\left(-\frac{(J_{ij})^2}{2J^2}\right) \qquad \text{and} \qquad \mathcal{P}(D_{ij}) = \frac{1}{D\sqrt{2\pi}}\exp\left(-\frac{(D_{ij})^2}{2D^2}\right). \tag{14}$$

To avoid overcounting, the summation over the component pairs (ab) is restricted by the condition $a < b$. The specific form of the magnetic Hamiltonian (13) may be motivated by physical systems in which the DM interaction is generated by both the spin–orbit coupling of the magnetic ions and the structural distortions of the lattice [26], which cause anisotropy of the DM interaction and correlation between its components.

After integration of the random couplings, introducing the spin-glass fields $Q_{iab}^{\alpha\beta}$ as above, and invoking the probability distribution of the Q-fields (4), the corresponding replicated partition function reads

$$\begin{aligned}\langle Z^{n_r}\rangle = \int \mathcal{D}Q\ \exp\Bigg\{&\sum_{\substack{ab\\ \alpha<\beta}}\Big[-\tfrac{1}{2}\sum_i (Q_{iab}^{\alpha\beta})^2 + (\beta J)^2\Gamma_{aa}\Gamma_{bb}\sum_{\langle i,j\rangle} Q_{iab}^{\alpha\beta}Q_{jab}^{\alpha\beta}\Big]\\ &+(\beta D)^2\sum_{\substack{\langle i,j\rangle\\ \alpha<\beta}}\Big[\sum_{(ab)} d_{ab}\ (Q_{iaa}^{\alpha\beta}Q_{jbb}^{\alpha\beta} - Q_{iab}^{\alpha\beta}Q_{jba}^{\alpha\beta})\\ &+\sum_{(ab)\neq(cd)} d_{ab}d_{cd}\ (Q_{iac}^{\alpha\beta}Q_{jbd}^{\alpha\beta} - Q_{ibc}^{\alpha\beta}Q_{jad}^{\alpha\beta}) + (i\leftrightarrow j)\Big]\\ &+\sum_{\substack{abc\\ \alpha<\beta<\gamma}}\sum_i Q_{iab}^{\alpha\beta}Q_{ibc}^{\beta\gamma}Q_{ica}^{\gamma\alpha} + \mathcal{O}(Q_i^4)\Bigg\}.\end{aligned} \tag{15}$$

To extract the soft modes of this action, we again concentrate on the quadratic terms and transform to the momentum representation. The eigenvalues of the resulting inverse zero-momentum propagator matrix $\boldsymbol{M}$ and their degeneracies are then given by

$$\begin{aligned}&\lambda_1 = 1 - z(\beta J)^2 && [(n-1)^2 - 4m^2]\text{-fold degenerate}\\ &\lambda_2 = 1 - z\beta^2(J^2 + D^2) && 2m^2\text{-fold degenerate}\\ &\lambda_3 = 1 - z\beta^2(J^2 - D^2) && 2m^2\text{-fold degenerate}\\ &\lambda_4 = 1 - z(\beta J)^2(1+\gamma)^2 && \text{non-degenerate}\\ &\lambda_5 = 1 - z(\beta J)^2(1+\gamma) && 2(n-1)\text{-fold degenerate}.\end{aligned} \tag{16}$$

According to the discussion given above, if inequality (9) holds, the smallest eigenvalue is λ_2, and the critical behaviour is thus controlled by a $2m^2$-component field whose components do *not* decouple. To elucidate the structure of the resulting field theory more clearly, and since new physical behaviour is already present for $m = 2$, in the following we consider the simplest case of *two* correlated separated DM couplings, which requires $n \geqslant 4$. Denoting the two correlated pairs of spin components by $(ab) = (12)$ and $(cd) = (34)$, we find the corresponding eight critical fluctuation modes

$$\begin{aligned} \phi_1 &= (\zeta/2)(Q_{11} + Q_{22} + Q_{33} + Q_{44}) & \phi_2 &= (\zeta/2)(Q_{11} + Q_{22} - Q_{33} - Q_{44}) \\ \phi_3 &= (\zeta/2)(Q_{12} - Q_{21} + Q_{34} - Q_{43}) & \phi_4 &= (\zeta/2)(Q_{12} - Q_{21} - Q_{34} + Q_{43}) \\ \phi_5 &= (\zeta/2)(Q_{13} + Q_{24} + Q_{31} + Q_{42}) & \phi_6 &= (\zeta/2)(Q_{13} + Q_{24} - Q_{31} - Q_{42}) \\ \phi_7 &= (\zeta/2)(Q_{14} - Q_{23} + Q_{41} - Q_{32}) & \phi_8 &= (\zeta/2)(Q_{14} - Q_{23} - Q_{41} + Q_{32})\,. \end{aligned} \tag{17}$$

Due to the symmetry relation $Q_{ab}^{\alpha\beta} = Q_{ba}^{\beta\alpha}$, the fields ϕ_1, ϕ_2, ϕ_5 and ϕ_7 are symmetric under permutation of replica indices, $\phi_s^{\beta\alpha} = +\phi_s^{\alpha\beta}$, whereas the fields ϕ_3, ϕ_4, ϕ_6 and ϕ_8 are antisymmetric, $\phi_a^{\beta\alpha} = -\phi_a^{\alpha\beta}$. From this property the replica structure of the corresponding propagators immediately follows:

$$\langle\phi_s^{\alpha\beta}(q)\phi_s^{\alpha\beta}(-q)\rangle = \langle\phi_s^{\alpha\beta}(q)\phi_s^{\beta\alpha}(-q)\rangle = \langle\phi_a^{\alpha\beta}(q)\phi_a^{\alpha\beta}(-q)\rangle = -\langle\phi_a^{\alpha\beta}(q)\phi_a^{\beta\alpha}(-q)\rangle \tag{18}$$

where $\alpha \neq \beta$. We now evaluate the contribution to the action cubic in the Q-fields and retain only the terms which couple to the eight soft modes,

$$\mathrm{Tr}\,\boldsymbol{Q}^3 = \sum_{\substack{abc\\ \alpha\beta\gamma}} Q_{ab}^{\alpha\beta} Q_{bc}^{\beta\gamma} Q_{ca}^{\gamma\alpha} \simeq \sum_{\alpha\beta\gamma} \sum_{i,j,k=1}^{8} u_{ijk}\phi_i^{\alpha\beta}\phi_j^{\beta\gamma}\phi_k^{\gamma\alpha}\,. \tag{19}$$

Of the 512 components of the interaction tensor u_{ijk} only 64 are non-zero; these are listed in table 1. With the full momentum dependence retained in (15), we then obtain the effective

Table 1. The 64 non-zero components of the tensor u_{ijk} for the n-vector spin glass with two correlated DM couplings. The table lists (ijk) and the sign of $u_{ijk} = \pm 1$.

(111)	(122)	(133)	(144)	(155)	(166)	(177)	(188)
+	+	−	−	+	−	+	−
(212)	(221)	(234)	(243)	(256)	(265)	(278)	(287)
+	+	−	−	−	+	−	+
(313)	(324)	(331)	(342)	(358)	(367)	(376)	(385)
−	−	−	−	−	+	+	−
(414)	(423)	(432)	(441)	(457)	(468)	(475)	(486)
−	−	−	−	+	−	−	+
(515)	(526)	(538)	(547)	(551)	(562)	(574)	(583)
+	+	−	−	+	−	+	−
(616)	(625)	(637)	(648)	(652)	(661)	(673)	(684)
−	−	+	+	+	−	+	−
(717)	(728)	(736)	(745)	(754)	(763)	(771)	(782)
+	+	+	+	−	+	+	−
(818)	(827)	(835)	(846)	(853)	(864)	(872)	(881)
−	−	−	−	−	+	+	−

Hamiltonian of the spin glass with two correlated DM couplings,

$$\mathcal{H}(\{\phi_l\}_l) = \tfrac{1}{2} \sum_{\alpha\neq\beta} \sum_{l=1}^{8} \phi_l^{\alpha\beta}(r - \nabla^2)\phi_l^{\alpha\beta} + w_0 \sum_{\alpha\beta\gamma} \sum_{i,j,k=1}^{8} u_{ijk}\phi_i^{\alpha\beta}\phi_j^{\beta\gamma}\phi_k^{\gamma\alpha} . \tag{20}$$

This Hamiltonian describes a cubic field theory of the general form studied by de Alcantara Bonfim *et al* [27]. To find its critical exponents, we have to evaluate the contractions of the tensor u_{ijk} as defined in [27]. To order $\mathcal{O}(\epsilon^2)$ it is sufficient to calculate the tensor contractions corresponding to the diagrams of figure 1. We find

$$\alpha = 16(n_r - 2) \qquad \beta = 8(n_r - 3) \qquad \gamma = 32 . \tag{21}$$

Taking the replica limit $n_r \to 0$ and inserting these results for α, β, γ in the general expressions of [27], with $\epsilon = 6 - d$ the critical exponents η and ν are finally obtained,

$$\eta = -\frac{\epsilon}{6} - \frac{7\epsilon^2}{288} \qquad \text{and} \qquad \nu = \frac{1}{2} + \frac{5\epsilon}{24} - \frac{13\epsilon^2}{1152} . \tag{22}$$

Up to order $\mathcal{O}(\epsilon)$ the exponents (22) agree with those of the random-gauge XY model [20] and of the n-vector spin glass in the $n \to \infty$ limit [25]. The coefficient of ϵ^2 is different from the random-gauge XY model [21] and from any isotropic n-vector spin glass [28, 29]. Thus, the spin glass with two correlated couplings of the random DM interaction is in a new universality class.

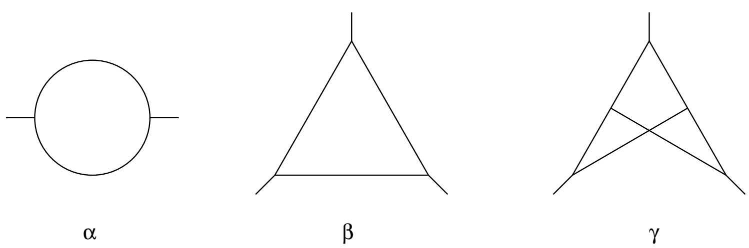

Figure 1. The tensor contractions α, β, and γ contributing at two-loop order as defined in [27].

4. The random-gauge XY model

To establish a closer relation between the results of the preceding sections and the field-theoretic treatment of the gauge glass, in this section we present a derivation of the effective field theory for the random-gauge XY model which is simpler and more rigorous than the earlier derivations [22, 23]. The random-gauge XY model is given by the Hamiltonian

$$\mathcal{H} = \sum_{\langle i,j\rangle} K_{ij} \cos(\phi_i - \phi_j + A_{ij}) \tag{23}$$

with the local phase ϕ_i and the quenched random-gauge factor A_{ij} which represents a phase twist along the bond $\langle i, j\rangle$ of the lattice which we assume to be a regular one. Defining a two-component spin vector $\boldsymbol{S}_i = \{\cos\phi_i, \sin\phi_i\}$ leads to the well known mapping on a planar spin Hamiltonian,

$$\mathcal{H} = \sum_{\langle i,j\rangle} \big(J_{ij}(\boldsymbol{S}_i \cdot \boldsymbol{S}_j) + D_{ij}\hat{\boldsymbol{z}} \cdot (\boldsymbol{S}_i \times \boldsymbol{S}_j)\big) . \tag{24}$$

The random spin couplings J_{ij} and D_{ij} are not independent, but are related via the gauge factor

$$J_{ij} = K \cos A_{ij} \qquad \text{and} \qquad D_{ij} = K \sin A_{ij} \tag{25}$$

where we have taken $K_{ij} = K$ independent of the bond. It has been shown by Gingras [18] in a real-space renormalization-group study that the random-gauge XY model (23) and the spin-glass model (24) with *independent* random couplings J_{ij} and D_{ij} are in the same universality class and thus deviations from the constraint (25) are irrelevant. Subsequently, the mapping of the XY spin glass with random DM interaction and independent couplings on a cubic two-component field theory has been established [23]. Although this field-theoretic analysis captures the correct critical behaviour, due to the neglect of higher cumulants it is strictly valid only for a Gaussian distribution of the couplings. On the other hand, the random-gauge XY model with uniformly distributed gauge phases permits an exact treatment of the disorder average which points out more clearly the analogy with our above results and which we present in this section.

We start with Hamiltonian (24) and choose uniform distributions for the local gauge phases A_{ij},

$$\mathcal{P}(A_{ij}) = \frac{1}{2\pi} \tag{26}$$

but take into account the condition (25). This leads to the replicated partition function

$$\langle Z^{n_r} \rangle = \mathrm{Tr}_{\{S\}} \prod_{\langle i,j \rangle} \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\varphi \ \exp(p_{ij} \cos\varphi + q_{ij} \sin\varphi) = \mathrm{Tr}_{\{S\}} \prod_{\langle i,j \rangle} I_0\left(\sqrt{p_{ij}^2 + q_{ij}^2}\right) \tag{27}$$

where

$$p_{ij} = \beta K \sum_{a\alpha} S_{ia}^{\alpha} S_{ja}^{\alpha} \qquad \text{and} \qquad q_{ij} = \beta K \sum_{ab\alpha} \varepsilon_{abz} \ S_{ia}^{\alpha} S_{jb}^{\alpha} \tag{28}$$

and I_0 is the modified Bessel function. Here ε_{abz} is the totally antisymmetric tensor and the summation over spin components a, b extends over x, y. Introducing the spin-glass fields $Q_{iab}^{\alpha\beta}$ and defining

$$\mathcal{Q}_{ij} = 2 \sum_{\substack{\alpha<\beta \\ ab}} \left(Q_{iab}^{\alpha\beta} Q_{jab}^{\alpha\beta} + Q_{iaa}^{\alpha\beta} Q_{jbb}^{\alpha\beta} - Q_{iab}^{\alpha\beta} Q_{jba}^{\alpha\beta} \right) \tag{29}$$

the partition function can be written in the form

$$\langle Z^{n_r} \rangle = \int \mathcal{D}Q \ \mathcal{P}(\{Q\}) \exp\left\{ \sum_{\langle i,j \rangle} \ln I_0(\beta K \sqrt{1 + \mathcal{Q}_{ij}}) \right\}. \tag{30}$$

We now expand the action in powers of $\mathcal{Q}_{ij}$ and omit constant terms and those of order $\mathcal{O}(Q_i^4)$ or higher to obtain

$$\langle Z^{n_r} \rangle = \int \mathcal{D}Q \ \mathcal{P}(\{Q\}) \exp\left\{ \sum_{\langle i,j \rangle} \left(\frac{\beta K}{2} \frac{I_0'(\beta K)}{I_0(\beta K)} \right) \mathcal{Q}_{ij} + \mathcal{O}(\mathcal{Q}_{ij}^2) \right\}. \tag{31}$$

Again, writing the zero-momentum contributions bilinear in the Q-fields in the form (7), one finds the eigenvalues of the inverse propagator

$$\begin{aligned} &\lambda_1 = 1 && \text{two-fold degenerate} \\ &\lambda_2 = 1 - 2z\beta K \ I_0'(\beta K)/I_0(\beta K) && \text{two-fold degenerate.} \end{aligned} \tag{32}$$

From this result, and if we identify the two critical modes by setting $a = x$ and $b = y$ in (10), the effective Hamiltonian (12) with $m = 1$ finally follows.

5. Conclusion

In the present paper the critical properties of classical spin glasses with anisotropic random Dzyaloshinskii–Moriya interactions have been addressed. We have found that the n-vector spin glass with $m \leqslant n/2$ independent and separated DM couplings is in the same universality class as the XY gauge glass. New physical behaviour is present in a spin glass with correlation between the separated random DM components. The explicit calculation of the critical exponents at two-loop level of an ϵ-expansion around six dimensions shows that such a spin-glass model with two correlated DM couplings belongs to a new universality class. Finally, in this paper we have presented a simplified and more rigorous field-theoretic treatment of the XY gauge-glass model.

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